

Study of BCS-BEC Crossover Physics and Evaluation of Energy Gap Parameters and Chemical Potential from BCS-theory

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(Received on : April 11, 2012)

ABSTRACT

Using the theoretical formalism of C. Regal *et al.* PRL (2004), we have studied the BCS-BEC crossover regime. We have evaluated the value of energy gap parameter and chemical potential for the crossover as a function of dimensionless parameters $|K_F a_s|^{-1}$ from BCS-theory. Our theoretical result indicates the crossover occurs in the very small region of the parameter $-1 < |K_F a_s|^{-1} < 1$.

Keywords: BCS-BEC crossover, Feshbach resonance, Pseudo-gap, chemical potential, Energy gap parameter, evaporative cooling.

1. INTRODUCTION

The techniques used to create alkali BEC were applied to other class of quantum particle, fermions. Earlier alkali atoms such as ⁸⁷Rb (composite bosons) were cooled as a gas down to nanokelvin temperature via laser cooling and evaporative cooling.¹⁻³ At these temperature the thermal de-Broglie wavelength of the particles becomes in order of the inter particle spacing in the gas and Bose Einstein condensate is formed. Experiments also observed that condensate

behave as matter wave⁴ and verified the superfluid nature of the condensates.^{5,6}

To create a Fermi gas of atoms, experiments applied the same cooling technique as those used to achieve BEC in ⁸⁷Rb or ²³Na. For Fermi gas of atoms, one has two stable alkali atoms ⁴⁰K and ⁶Li with an odd number of electron, protons, and neutrons. The first gas of fermionic atoms to enter the quantum degenerate regime was created at JLLA⁷ in 1999 using ⁴⁰K. The observation in this experiment was not a phase transition as in the case of Bose gas

but rather the presence of more and more energy that would be expected classically as the Fermi gas was cooled below the Fermi temperature. Many more Fermi gas experiments using a variety of cooling techniques then followed.⁸⁻¹⁶

The next goal after the creation of a normal Fermi gas of atoms was to form a superfluid in a paired Fermi gas. In conventional superconductors s-wave pairing occurs between spin-up and spin-down electrons. The hope was that s-wave pairing could similarly occur with the creation of two components atomic gas with an equal Fermi energy for each components. Such a two- component gas can be realized using an equal mixture of alkali atoms in two different hyperfine spin state. The idea was that BCS state would appear if the temperature of this two component gas were cold enough and the interaction between fermions attractive and large enough. However, for typical interactions the temperature required to reach a true BCS state were far too low compared to achievable temperature to imagine creating cooper pairs. Stoof *et al.*¹⁷ noted that the interaction between ⁶Li atoms were large compared to typical values of the scattering length ($|a_s|=2000 a_0$) as well as attractive bringing the BCS transition temperature closer to the realistic temperature.^{18,19} It was then realized that a type of scattering resonance known as Feshbach resonance could allow arbitrary changes in the interaction strength. Theories were developed that explicitly treated the case where the interactions were enhanced by Feshbach resonance.^{20,21} In a theory, it was proposed that the BCS wave functions was more generally applicable to the weakly

interacting limit. As long as the chemical potential is found self consistently (as the interaction is increased the BCS ground state) can describe everything from Cooper pairing to the BEC of composite boson made up of two fermions. After nearly a century of ⁴He and superconductor being considered as separate entities that experimental realization of superfluid in BCS-BEC crossover regime would provide a physical link between the two.²² More recent interest in crossover theories has also come in response to the possibility that they could apply to high T_c superconductors. These superconductors differ from normal superconductors both in their high transition temperature and the apparent presence of the pseudo gap, which are both characteristics expected to be found in a Fermi gas in the crossover.^{23,24}

In this paper, we have studied the BCS-BEC crossover physics using BCS theory. BCS theory was originally applied in the limit where the interaction energy is extremely small compared to the Fermi energy. In this case, the chemical potential μ can be fixed at E_F and many calculations become reasonable simple. Leggett²⁵ pointed out that if the BCS gap equation is examined allowing μ to vary, the gap equation actually becomes precisely the Schrodinger equation for diatomic molecule in the limit where μ dominates. The structure of the crossover theory originates with the work of Nozieres and Schmitt-Rink²⁶ and Randeria *et al.*²⁷

2 MATHEMATICAL FORMULA USED IN THE STUDY

Let us consider a homogeneous Fermi system in three dimensions in an

equal mixture of two different states at $T=0$ Applications of usual BCS theory²⁸ results in the gap equation

$$\Delta_k = - \sum_{k'} U_{kk'} \frac{\Delta_{k'}}{2E_{k'}} \quad (1)$$

where

$$E_k = \sqrt{(\epsilon_k)^2 + \Delta^2}$$

$$\epsilon_k = \epsilon_k - \mu \text{ and } \epsilon_k = \hbar^2 k^2 / 2m$$

$U_{kk'} < 0$ is the attractive interaction for scattering of fermions with momenta k' and $-k'$ to k and $-k$. Than one obtains the number equation

$$\langle N_{tot} \rangle = \sum_k (1 - (\epsilon_k / E_k)) \quad (2)$$

Where N_{tot} is the total number of fermions in both states. To solve equation (1) in the BCS limit the standard approach is to assume that the potential is constant at a value $U < 0$ which implies that gap is constant as well i.e $\Delta_k = \Delta$. In that case the gap equation becomes.

$$-\frac{1}{U} = \sum_k \frac{1}{2E_k} \quad (3)$$

Now one finds that this equation diverges. For BCS superconductor this problem is resolved because the interaction can be limited to within the Debye energy $\hbar\omega_D$ of E_F . This is the result of the nature of the phonon mediated interaction between the electrons that gives rise to the attractive interaction. Further simplification of BCS-

limit are that $\mu = E_F$ and that since $\hbar\omega_D \ll E_F$ the density of states is constant at the value N ($\xi = 0$). The gap equation then becomes

$$-\frac{1}{N(0)U} = \int_{-\hbar\omega_D}^{\hbar\omega_D} \frac{d\xi}{2[\Delta^2 + \xi^2]^{1/2}} \quad (4)$$

Solving eqⁿ (4) produces the BCS results

$$\Delta \cong 2\hbar\omega_D \exp\left[-\frac{1}{N(0)|U|}\right] \quad (5)$$

To extend this calculation to the crossover in atomic system one can no longer apply the $\hbar\omega_D$ cut off. The solution to the problem in this case is nontrivial and requires a renormalization procedure. M. Randeria²³ have obtained the result of such a procedure and obtained the renormalized gap equation.

$$\frac{-m}{4\pi\hbar^2 a_s} = \frac{1}{V} \sum_k \left(\frac{1}{2E_k} - \frac{1}{2\epsilon_k} \right) \quad (6)$$

where the interaction U is described by the s-wave scattering length a_s and V is the volume of the system. One can't assume $\mu = E_F$ in the crossover. For this, one solves gap equation given in equation (6) and number equation (2) simultaneously for μ and gap parameter Δ . One solves these parameter as a function of dimensionless parameter $(K_F a_s)$ where $K_F = (2mE_F)^{1/2}/\hbar$. M. Marim *etal.*²⁹ have done this analytically using the elliptical integrals. We have taken the help of this paper for calculating Δ and μ . The results are shown in table T₁ and T₂

for a homogeneous Fermi gas at $T = 0$ as determined through NSR theory.²⁶

Beyond $T = 0$

The phase transition temperature T_c is an important parameter for superfluid system. In BCS-BEC crossover the transition temperature T_c increases as the interaction is increased. It is lowest in the perturbative BCS regime and highest in the BEC limit. In a homogeneous system in the BCS limit.³⁰

$$\frac{T_c}{T_F} = \frac{8}{\pi} e^{\gamma-2} \exp\left[\frac{-\pi}{2K_F |a_s|}\right], \gamma = 0.58 \quad (7)$$

$$\text{In the BEC limit limit}^{31} T_c / T_F = 0.22 \quad (8)$$

The BCS transition temperature can be extremely small due to the exponential dependence upon $1/(K_F a_s)$. If one puts the interaction strength in the alkali fermionic gas ($a_s = -100 a_0$) and a typical K_F ($1/K_F = 200 a_0$), a_0 is Bohr radius.

$$T_c(\text{BCS}) = 10^{-14} T_F \quad (9a)$$

Which is completely inaccessible temperature in atomic systems.

$$\text{For } 1/(K_F a_s) = -1, T_c(\text{BCS}) = 0.1 T_F \quad (9b)$$

Which can be accessible.

In the BCS limit pairing and the phase transition to a superfluid state occur at the same temperature. However in the BEC limit this is not the case because the constituent fermions are very tightly bound pairs and can form far above T_c .

3. DISCUSSION OF RESULT

In this paper we have studied the BCS- BEC crossover physics from BCS theory. We have taken the theoretical formalism of C. Regal *et al.*^{32,33} in this study. In table T₁ and T₂ we have presented the evaluated result of the gap parameter Δ/E_F and chemical potential μ/E_F as fraction of dimensionless parameter $(K_F a_s)^{-1}$ determined through NSR theory for BCS and BEC limit. From our theoretical results it appears that crossover occurs in a relatively small region of the parameter $(K_F a_s)^{-1}$ from $-1 < (K_F a_s)^{-1} < 1$. In table T₃ we have given the value and meaning of both μ and Δ for the BCS and BEC limit. The recent theoretical result^{34,35} also confirm such behavior.

Table T₁

An evaluated results of the gap parameter Δ/E_F as a function of dimensionless parameter $(K_F a_s)^{-1}$ determined through NSR theory.

$(K_F a_s)^{-1}$	(Δ/E_F)	
	BCS Limit	BEC Limit
2.5	2.85	0.00
2.0	2.53	0.00
1.5	2.10	0.00
1.0	1.82	0.00
0.5	1.02	0.00
0.0	0.00	0.00
-0.5	0.00	1.27
-1.0	0.00	1.13
-1.5	0.00	0.87
-2.0	0.00	0.42
-2.5	0.00	0.32
-3.0	0.00	0.00

Table T₂

An evaluated results of the chemical potential (μ/E_F) as a function of dimensionless parameter ($K_F a_s$)⁻¹ determined through NSR theory.

$(K_F a_s)^{-1}$	(μ/E_F)	
	BCS Limit	BEC Limit
-2.5	1.122	0.00
-2.0	1.120	0.00
-1.5	1.116	0.00
-1.0	1.115	0.00
-0.5	1.115	0.00
0.0	1.115	0.00
0.5	0.00	0.085
1.0	0.00	-1.052
1.5	0.00	-2.386
2.0	0.00	-3.458
2.5	0.00	-5.687
3.0	0.00	-6.432

Table T₃

Crossover experiment have been performed with ⁴⁰K and ⁶Li. The regime corresponds to varying a_s from -2000 a_0 through ∞ and to 2000 a_0 . a_0 is Bohr-radius. $a_0 = 0.0529$ nm. Now the value and meaning of both μ and Δ are different in two limits.

BCS Limit	BEC Limit
$\mu \sim E_F$	$\mu \sim \frac{-E_b}{2} = -\left(\frac{1}{K_F a_s}\right)^2 E_F$
$\Delta \sim e^{-\pi/K_F a_s E_F}$	$\Delta \sim \left[\frac{16}{3\pi} \frac{1}{K_F a_s}\right]^{1/2} E_F$

Δ is the gap parameter but its meaning is the excitation gap i.e. the smallest possible energy that can create a hole (remove fermions) in the superfluid in the BCS limit.

In general the excitation energy is

$$E_{gap} = m_{\min} E_k = m_{\min} [(\epsilon_k - \mu)^2 + \Delta^2]$$

$$= m_{\min} \left[\left(\frac{\hbar^2 K^2}{2m} - \mu \right)^2 + \Delta^2 \right]$$

Δ is positive when μ is positive (BCS limit) but becomes $[\mu^2 + \Delta^2]^{1/2}$ when μ is negative.

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